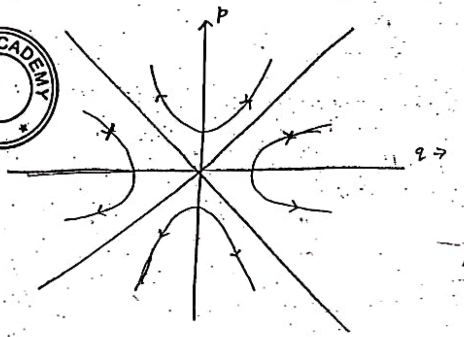
**Prob:** The Hamiltonian of a particle is given by

$$H = \frac{p^2}{2m} - \frac{\alpha z^2}{2}$$

which of the following fig represents the motion of the particle in phase space.

$$H = \frac{p^2}{2m} - \frac{\alpha z^2}{2} \qquad E \rightarrow \text{constt.}$$

P.S-digm: Parabola $\qquad V(x) = -\frac{\alpha z^2}{2}$ i.e. eqn of para




$$\dot{q} = \frac{\partial H}{\partial p} = p/m$$

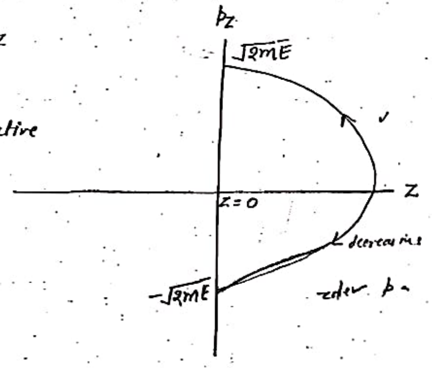
$$\dot{p} = -\frac{\partial H}{\partial q} = \alpha q$$

* The trajectory (Phase space diagram) on $z-p_z$ plan of ball bouncing from a perfect elastic hard sphere.

$$H = \frac{p^2}{2m} + mgz$$

$E$ = constt

system is conservative




→ If the particle is bouncing from inelastic sphere :

Eleastic

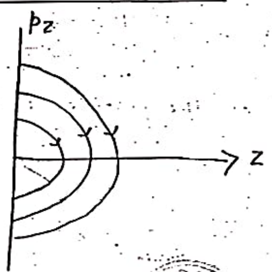
$P = P'$ $\quad T = T'$

$\vec{P} \neq \vec{P}'$

$P \neq P'$ Inelastic

$\vec{P} \neq \vec{P}'$

$T = T'$



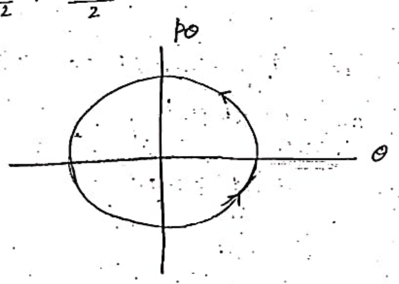

→ Phase space diagram of Simple pendulum :

$$H = \frac{p_\theta^2}{2ml^2} + mgl(1-\cos\theta)$$

$\theta$ is small

$\cos\theta = 1 - \theta^2/2$

$$H = \frac{p_\theta^2}{2ml^2} + \frac{mgl}{2}\theta^2$$

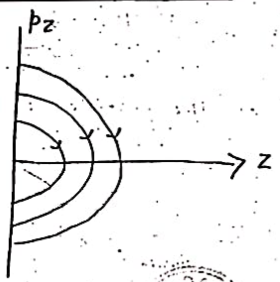



$$E = \frac{p_x^2}{\ldots} + \frac{1}{\ldots} K x^2$$

→ If the particle is bouncing from inelastic sphere :

Elastic
$P = P'$, $T = T'$
$\vec{P} \neq \vec{P}'$

$P \neq P'$ Inelastic
$\vec{P} \neq \vec{P}'$
$T = T'$

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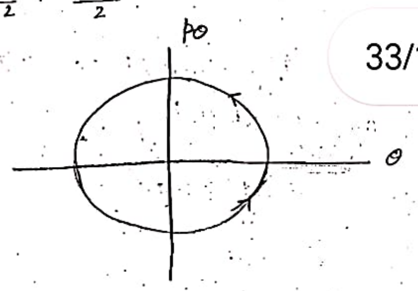
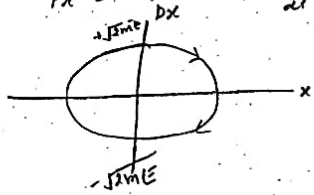
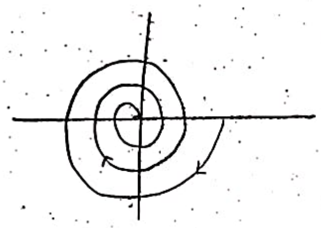



$$E = \frac{p_x^2}{2m} + \frac{1}{2}Kx^2$$

$\dot{p}_x = -Kx \Rightarrow \frac{dp_x}{dt} = -Kx$




→ If bob is immersed in water then phase space diagram



* Boltzmann's Definition of Entropy →

$$S = K_B \ln\Omega$$

Consider two independent system having macrostates are described by $(N_1, V_1, E_1)$ & $(N_2, V_2, E_2)$ with thermodyna. probabilities $\Omega_1(N_1, V_1, E_1)$ & $\Omega_2(N_2, V_2, E_2)$ respectively.

| | |
|---|---|
| $\Omega_1(N_1, V_1, E_1)$ | $\Omega_2(N_2, V_2, E_2)$ |

Thermodynamical probability of composition

$\Omega = \Omega_1 \times \Omega_2$ —①, En. of composition $E = E_1 + E_2$ —②

If energy is allowed to exchange

$\Omega = \Omega_1(E_1) \times \Omega(E_2)$

Let equilibrium energy is $\bar{E}$

$\Omega$ = max